Mobility and local income redistribution

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Abstract

Mobility may undermine local income redistribution in federal systems, because rich taxpayers can evade high taxes by moving to low tax jurisdictions. By analyzing a model of local income redistribution with endogenous voting, income heterogeneity and an exogenously given degree of mobility we focus explicitly on the link between redistribution and mobility. Our findings suggest a nonlinear relationship between redistribution and mobility: high and low degrees of mobility permit major income redistribution as income sorting is absent, while a medium degree of mobility leads to high differences in tax rates between jurisdictions and thus to income sorting and less redistribution. Moreover, a high level of overall inequality leads to income sorting regardless of the degree of mobility. Our results indicate that immobile rich in high tax jurisdictions and immobile poor in low tax jurisdictions suffer most from income sorting.

Key words: Redistribution, Political economy, Locational equilibrium, Taxes, Tax havens.

JEL classification: H23, H71, H73.

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1 Introduction

Mobility is often considered as constraining local income redistribution in a federal system. The tax competition literature generally argues that local redistribution is impossible or at least difficult to implement if the population is mobile, as rich taxpayers evade high tax jurisdictions by moving to low tax jurisdictions. Nevertheless, the majority of high-income countries are organized as federations, where subnational jurisdictions are endowed with a certain level of autonomy concerning local taxes and redistribution.

In this paper, we ask which factors play a role for the functioning of local redistribution and whether a substantial amount of local redistribution is feasible despite the mobility of taxpayers. To this aim we analyze an equilibrium model of local income taxation, where taxpayers differ with respect to income and their degree of mobility. Part of the population can move freely between jurisdictions. Taxes are determined endogenously by voting and are used for redistribution in form of a lump-sum grant to every taxpayer. Thus, our model incorporates two equilibrium concepts: a locational and a political equilibrium. In the locational equilibrium every taxpayer is content with the tax-grant package in the chosen jurisdiction and does not want to move. In the political equilibrium tax rates are determined by inequality (median to mean ratio) in the jurisdiction. These two equilibrium concepts have been found to produce so called "income stratification" patterns (see e.g. Epple and Romer (1991), Hansen and Kessler (2001) or Schmidheiny (2006b)), which means that taxpayers sort themselves into jurisdictions according to income. In a stratified equilibrium taxpayers with the highest income live in low tax jurisdictions, taxpayers with middle income in medium tax jurisdictions, etc.

Stratification does not mean that there is no redistribution at all. Differences between tax rates and thus redistribution arise endogenously through voting. Depending on how large the differences between jurisdictional tax rates become, the less redistribution is feasible between the rich and the poor.

We contribute to the growing literature on stratification by introducing the degree of mobility explicitly into the redistribution model. This allows us to determine the influence of the degree of mobility on income stratification and thus on redistribution.
Furthermore, the model helps to show which groups of individuals may or may not profit from income stratification. Our analysis yields three important results.

First, it predicts that if the general level of mobility is low, stratification cannot arise because rich taxpayers will be too unequal between each other relative to the group of poorer taxpayers. In this case voting does not lead to a lower tax rate in the jurisdiction where the rich live than elsewhere. This result is in the spirit of Hansen and Kessler (2001) who argue that tax havens only arise in small countries as there is less inequality between the super rich than between super rich and medium rich.

Second, we find that if only part of the population is immobile, income stratification arises. The intuition is that there is more inequality and thus redistribution in the jurisdiction where the poor decide to live when some rich individuals are unable to move. The immobile rich are exploited by the poor mobile taxpayers in the high tax jurisdiction. The groups that profit from this kind of income stratification are mobile individuals, but also the rich immobile in the low tax jurisdiction and the poor immobile in the high tax jurisdiction. The rich immobile in the high tax jurisdiction loose when compared to a situation where everyone is mobile because they now have to pay higher taxes which they cannot escape. The poor immobile living in the low tax jurisdiction also loose because they obtain lower grants and have to pay higher rents. Note that very high levels of immobility logically lead to less income stratification as only few people move so that tax rates become rather similar. Thus our model predicts a non-linear (hump shaped) pattern between mobility and income stratification which may help to explain empirically often insignificant results and signs when analyzing the relationship between inequality and redistribution (see Bénabou (2002) for a review).

Third, we analyze the influence of existing aggregate income inequality in the regional economy (a region consisting of several separate jurisdictions) on the income stratification between local jurisdictions. We find that if aggregate inequality is high there is always a stratification outcome. Intuitively, a higher aggregate income inequality increases ex-ante differences between jurisdictions and makes stratification more likely for every degree of mobility.

These three theoretical results suggest that local income redistribution in a federal
system is only effective if aggregate income inequality is not too high and mobility is either relatively high or relatively low.

The paper is structured as follows. The next section discusses how our contribution is related to the existing literature. Section 3 describes the model of local redistribution. In Section 4 we formally analyze the equilibrium of the model and derive the main results. Section 5 concludes.

2 Related Literature

In his seminal paper Tiebout (1956) (later formalized by Ellickson (1971) and criticized by Epple and Zelenitz (1981)) argues that a decentralized system of taxation and public goods provision allows citizens to "shop among jurisdictions" according to their preference. A phenomenon that is known as "Tiebout sorting". Empirically, Oates (1969) finds evidence for Tiebout sorting, whereas Pollakowski (1973) shows that his initial results are not very robust. More recently Rhode and Strumpf (2003) find that heterogeneity of public goods across communities decreases over time which is against the predictions of an extended Tiebout model. The later theoretical literature therefore deviates explicitly from the assumptions of the Tiebout model to test its robustness (see e.g. Tiebout-like models incorporating an explicit voting process like Westhoff (1977), Westhoff (1979) or with an explicit land market like Rose-Ackerman (1979) and Bucovetsky (1982)).

More recently, it has become quite standard to combine an explicit voting process with a migrational decision à la Tiebout to obtain a general equilibrium approach. In this approach, individuals typically move to a preferred jurisdiction in a first stage and there is majority voting over fiscal policy and taxes in a second stage. When moving individuals foresee the outcome of majority voting in the second stage, such that once an equilibrium is reached, they have no further incentive to move. The present paper follows such an approach and we consider a model of income taxation. In contrast, a lot of authors analyze models of property taxation which is the dominant form of local public finance in the US (see Epple, Filimon, and Romer (1984), Epple, Filimon, and Romer (1993), Nechyba (1997) or Epple and Platt (1998)). Epple and Romer (1991)
analyze the influence of a special aspect of mobility in a setting with property taxation. More specifically they differentiate between two main cases: (1) a model where there are only renters who rent houses from absentee landlords and (2) a model with homeowners who consider also the capital gains or losses they will incur as a result of a change in the net-of-tax price of housing induced by a change in the level of redistributive taxation. The conditions for the existence of an equilibrium of the model are the same in both cases, but an owner with a given endowed income will prefer a lower level of redistributive taxation than a renter with the same income.

Despite the frequent focus on property taxation, there are some notable exceptions that analyze general equilibrium models of local public good provision with income taxation (see Goodspeed (1986), Goodspeed (1989), Hansen and Kessler (2001), Kessler and Lülfesmann (2005) and Schmidheiny (2006b)). Those contributions all have a slightly different focus to ours in the sense that they do not analyze the influence of different degrees of mobility on stratification. Goodspeed (1986) and Goodspeed (1989) derives the conditions for existence of an equilibrium in a metropolitan model with income taxation, congestible public goods and a housing market. He also estimates the welfare loss from changing from a head tax to a proportional income tax. However, he does not consider the influence of different mobility patterns. Hansen and Kessler (2001) focus on explaining the existence of tax havens depending on the size of a country (or jurisdiction). They show that if there is a jurisdiction of a very small size which can accommodate the most affluent individuals, a stratification equilibrium can arise. In Hansen and Kessler (2001) the decisive characteristic of a jurisdiction which leads to stratification of individuals with respect to income is the difference in country (or jurisdictional) size, whereas in our setting the fraction of the non-mobile population plays a similar crucial role. Alternatively, Kessler and Lülfesmann (2005) show that stratification equilibria can be due to different preferences for public goods. Finally, analyzing a richer model where individuals differ in both income and preferences for housing, Schmidheiny (2006b) derives imperfect income segregation. All these contributions, like ours, show the possible existence of stratification equilibria. However the approach presented in this paper is different as it is the fraction of immobile individuals which matters for stratification to occur. This
paper consequently extends existing work on income stratification equilibria in models with income taxation.

Similarly to Epple and Romer (1991) and Hansen and Kessler (2001) we analyze a purely redistributive setting without considering a public good which could enter the utility function explicitly. Grossmann (2002) argues that the distinction between a purely redistributive setting versus a public goods setting matters in models of majority voting because the consequences for the nature of the link between inequality and redistribution (or the size of government) differ between those settings. In a redistributive setting higher income inequality leads to more redistribution in equilibrium, whereas in a public goods setting a higher income inequality may lead to less redistribution. Empirically, the relationship between income inequality and redistribution has been investigated by many authors with rather inconclusive results (see, among others, Perotti (1994) or Persson and Tabellini (1994)). Bénabou (2002) presents a review of the results of this strand of literature and finds that "the results are rather disappointing: the effect of income distribution on transfers and taxes is rarely significant, and its sign varies from one study or even one specification to another."\(^1\) However, some studies find evidence in favor of the redistributive setting (see e.g. Milanovic (2000) or Perotti (1996)). Being aware of the criticism we still opt for a redistributive setting, as our main interest is to assess the implications of mobility on taxation and on opportunities for income redistribution via grants.

We look at jurisdictions in a metropolitan area centered around a big city, where all individuals go to work in the same place but live in different jurisdictions. Therefore, we neglect possible effects coming from local productivity differences which would be important when looking at jurisdictions on a higher level such as countries. (Productivity differences are taken into account in models of spatial asset pricing, see e.g. Ortalo-Magné and Prat (2008)). Furthermore, we look at a static model which does not allow for growth and human capital accumulation (effects which are analyzed in recent contributions such as Bénabou (1996a), Bénabou (1996b), Bénabou (1996c), Glomm and Lagunoff (1999), Fernández and Rogerson (1998) or Epple, Romano, and Sieg (2009)). Given our focus

on redistributive government expenditures we leave such considerations out as they are especially interesting when analyzing schooling expenditures and public investments.

One main implication of our model is that in equilibrium individuals may be stratified with respect to income classes. In the empirical literature we indeed find some evidence for income stratification patterns. Using data from US federal states, Bakija and Slemrod (2004) show that wealthy retirees try to avoid high state taxes by changing their state of residence. For the special case of Switzerland, where cantons and also communities enjoy a relatively high tax autonomy Pommerrehne, Kirchgässner, and Feld (1996) and Feld and Kirchgässner (2001) find evidence for some income stratification due to differences in taxes. However, Liebig and Sousa-Poza (2006) analyze micro data and do not find significant tax-induced migration in Switzerland. Brülhart and Jametti (2006) analyze the existence of vertical versus horizontal tax externalities and find that the former dominate the latter, which might be evidence for tax competition between communities and cantons. These studies deserve credit for showing that tax differences and stratification occur, when jurisdictions enjoy fiscal autonomy and they show to what extent empirically individuals react to such tax differences by migration. However, they assume that tax rates are set exogenously by a local government (benevolent or not). It is not a priori clear how such results have to be interpreted when taxes are set endogenously, as for example by a voting process where the residents have to decide for some political candidate who proposes a certain tax policy (as in the case of majority voting). Thus, more recently, a lot of authors attempted to structurally estimate or calibrate equilibrium models of local jurisdictions where voting is endogenous (see Epple and Sieg (1999), Epple, Romer, and Sieg (2001), Bayer, McMillan, and Rueben (2004), Bajari and Kahn (2005), Sieg, Smith, Banzhaf, and Walsh (2002), Sieg, Smith, Banzhaf, and Walsh (2004), Ferreyra (2007), Ferreira (2004) and for Switzerland Schmidheiny (2006a)). These contributions generally find strong evidence for income stratification between local jurisdictions.

The focus of our paper is on the influence of mobility patterns on taxation and redistribution. Therefore, we also contribute to the part of the literature which makes predictions about the influence of the degree of mobility on local taxation and the possi-
bility of redistributive policies at the local level. In the "race-to-the-bottom" literature a higher mobility typically leads to lower taxes and less redistribution (see Brueckner (2000)). Empirical studies analyzing whether this breakdown of the welfare state really occurs yield different results. Brown and Oates (1985) and Feldstein and Wrobel (1998) find evidence for tax avoidance in the US similar to the one presented above and thus argue that redistribution should be taken out by the central government only. However, for Switzerland Feld (2000) finds that a considerable amount of redistribution takes place at the local level despite high mobility of tax payers and strong tax competition. Alternatively, Lee (2007) presents a political-support approach to redistribution in a federation, which leads to a different conclusion than the standard "race to the bottom" models. He finds that depending on the cost of housing (which is equivalent to mobility costs in his model), mobility may increase or decrease income redistribution.

Our theoretical model helps to show which groups with different degrees of mobility may or may not profit from income stratification and our analysis predicts a non-linear pattern between mobility and income stratification (redistribution) which may serve as a possible explanation for rather inconsistent empirical results.

3 The Model

We look at a system of integrated and independent jurisdictions denoted \( j = 1, \ldots, J \) in an economy, all having the same size. The jurisdictions are politically independent in the sense that each jurisdiction determines fiscal policy in an autonomous way. We choose a setting similar to recent contributions on stratification such as Hansen and Kessler (2001) or Kessler and Lülfesmann (2005). But different from them, in our economy jurisdictions distinguish themselves only by the fraction of mobile individuals.

We develop a model of redistribution to analyze the interaction between mobility, the economy’s income inequality, and political decisions on redistributive taxes. The most important elements of the model are: (1) Individuals are heterogeneous with respect to income so that there is a redistributational issue. (2) A jurisdiction’s budget must balance. (3) Migration is costless for the group of mobile individuals, i.e. in equilibrium mobile
persons must be unable to improve their positions by moving. (4) After settlement in a specific jurisdictions, political decisions are determined by majority voting.

Denote the two groups of individuals in the economy by $M$ for mobile and $I$ for immobile. The part of the immobile population in the economy is expressed as $\lambda$. Each individual $i$, irrespective of its group membership, earns an exogenous income $y^i$ which can be interpreted as location independent labor income. Each individual has to rent a unit of housing from competitive absentee landlords to gain the right to live and vote in a jurisdiction. Rents in jurisdiction $j$ are denoted as $r_j$. Housing demand is normalized to unity. The utility $V$ of an individual $i$ is assumed to be linear in consumption $c^i$. Individuals in jurisdiction $j$ have to pay proportional income taxes $t_j$ on their income $y^i$ and receive a (basic income) grant $g_j$ from the local government.\(^2\) The budget constraint of individual $i$ in jurisdiction $j$ is thus

$$y^i + g_j = c^i + y^i t_j + r_j,$$

and indirect utility of individual $i$ is equal to net income minus the rent

$$V(y^i, t_j, g_j, r_j) = (1 - t_j)y^i + g_j - r_j.$$

In the economy income is distributed across all individuals according to the distribution function $F(y)$ with density $f(y) > 0$ and support $[0, \text{max } y]$. Individuals in each group ($M$) and ($I$) have the same distribution of income. In every jurisdiction, the income distribution for immobile individuals is given by $f^I(y)$. The income distribution for mobile individuals for each jurisdiction $j$ emerges endogenously after migration and is denoted $f^M_j(y)$. We normalize each jurisdiction’s size in the economy to unity. Consequently, $\lambda$ represents the percentage of immobile individuals ($I$) in each jurisdiction. From the viewpoint of mobile individuals, the part of immobile individuals $\lambda$ determines the "size" of the jurisdiction for them. Changes in average and median incomes de-

\(^2\)We might introduce local public good provision instead of pure redistribution. In this case the preference for public goods would play a role. Instead, we opt for a purely redistributive setting, because the focus of our paper is the influence of mobility and not the influence of different preferences for public goods.
pend on the part of the mobile as well as the immobile population. Therefore, in our setting, the part of immobile individuals has potentially different implications than a jurisdiction’s size in the model of Hansen and Kessler (2001). To insure that rents in each jurisdiction of the economy are fully determined even when the population size in each jurisdiction is fixed, we suppose that competitive absentee landlords underbid each other in the jurisdiction with the lowest mean income. More specifically, in the jurisdiction with the lowest mean income rents must equal the break-even price $r$ for absentee landlords.\(^3\)

To focus on political motives for government and individual behavior, we abstract from allocative reasons for public spending and assume that jurisdictions raise income taxes for redistributive purposes only. The government has a redistribution policy in the sense that it confers the same basic income or grant $g_j$ to every individual in the jurisdiction. Proportional income taxes $t_j$ finance the grant $g_j$. This "ability to pay principle" is a common approximation of the progressive tax systems in use. A jurisdiction’s budget must balance and thus the budget constraint is

$$
(t_j - \frac{1}{2} t^2_j) \int_0^{\max_y} ydF_j = g_j,
$$

where $F_j$ denotes the endogenous measure of individuals with income $y$ living in jurisdiction $j$ such that average income in jurisdiction $j$ can be expressed as $\bar{y}_j = \int_0^{\max_y} ydF_j$. The budget constraint equates local income tax revenues minus costs of raising public funds. The term $(t_j - \frac{1}{2} t^2_j)$ outlines a concave per capita Laffer-curve. Taxation is costly with costs taking the form $t^2_j/2$ for simplicity. We assume that the jurisdictions all have the same efficiency in redistribution, i.e. the same costs of taxation.\(^4\) Because of the binding budget constraint we can solve for the grant $g_j$, which is then determined by the

\(^3\)Technically we could also assume that there is an oversupply of houses.

\(^4\)As incomes are exogenous and pure redistribution is analyzed, the absence of costs of taxation would allow for arbitrarily high tax rates. Deadweight losses of distortive taxation could also be introduced more generally by endogenous labor supply. This would mainly complicate the analysis without changing the qualitative results and providing additional insights.
chosen tax rate

\[ g_j = \left( t_j - \frac{1}{2} t_j^2 \right) \bar{y}_j. \]  

(1a)

4 Equilibrium Analysis

The equilibrium of the model is determined in two steps:

1. Mobile individuals choose the jurisdiction to live in which results in the locational equilibrium.

2. All individuals in a jurisdiction cast their votes which results in the voting equilibrium.

When backward solving the model we look at the voting equilibrium first. Given the conditions resulting from the voting equilibrium we analyze the locational equilibrium.

Every voter \(i\) maximizes his indirect utility with respect to the tax rate and the basic income grant given the budget constraint, i.e.

\[ \max_t (1 - t_j) y^i + (t_j - \frac{1}{2} t_j^2) \bar{y}_j - r_j \]

Solving the individual maximization problem for tax rates leads to \(t_j = 1 - \frac{y_j}{\bar{y}_j}\). In the unique majority voting equilibrium the tax choice of the median voter is implemented, i.e.

\[ t_j^* = 1 - \frac{y_j^m}{\bar{y}_j}, \]  

(2)

and the level of redistribution in equilibrium is given by \(g_j^* = \frac{1}{2} \frac{\bar{y}_j^2 - (y_j^m)^2}{\bar{y}_j}\). Redistribution will be higher in jurisdictions with high mean income and lower in jurisdictions with a high median income. The political choice by the median voter determines the fiscal package of taxes and grants \((t_j^*, g_j^*)\).

Given the equilibrium tax rate we can determine the locational equilibrium of how the population locates over jurisdictions. In a locational equilibrium a mobile individual
individual \( i \) must be indifferent between jurisdictions. If individual \( i \) decides to settle in jurisdiction \( j \) his/her utility must be greater or equal in jurisdictions \( j \) than in any other jurisdiction denoted \(-j\), i.e. \( V(y^i, t_j, g_j, r_j) \geq V(y^i, t_{-j}, g_{-j}, r_{-j}) \). More precisely:

**Definition 1** An equilibrium in the economy is defined as an income distribution of mobile individuals in each jurisdiction \( f_j^M(y) \), a fiscal policy package \((t^*_j, g^*_j)\) and rental fees \( r^*_j \) for all \( j = 1, \ldots, J \), such that (1) no mobile individual with income \( y^i \) living in \( j \) has an incentive to move to another jurisdiction \(-j\), that is, \( V(y^i, t^*_j, g^*_j, r^*_j) \geq V(y^i, t_{-j}, g_{-j}, r_{-j}) \) and (2) tax rates \( t^*_j = 1 - \frac{y_m}{\bar{y}_j} \) reflect the median voters choice in each jurisdiction.

### 4.1 A symmetric equilibrium

First, we show that independent of the part of immobile individuals \( \lambda \), there always exists a symmetric equilibrium in which all jurisdictions implement the same fiscal policy package, \((t_j, g_j)\), and rents, \( r_j \), as well as average incomes, \( \bar{y}_j \) are the same across all jurisdictions.

Suppose that the population is distributed symmetrically over all jurisdictions in the economy such that in each jurisdiction the local income distribution of mobile individuals is equal, i.e. \( f_j^M(y) = f^M(y) \) for all \( j = 1, \ldots, J \). This is equivalent to saying that median and average incomes are the same in all jurisdictions, independent of the part of the population that is immobile \( \lambda \). Therefore, tax rates and consequently also the level of redistribution will be the same in each jurisdiction. The clearing of renting markets implies that absentee landlords will set the same renting price denoted by \( \bar{r} \). Therefore every mobile individual is indifferent between jurisdictions and thus the presumed distribution \( f^M(y) \) is an equilibrium outcome. The following Proposition summarizes this finding (all proofs are relegated to the Appendix):

**Proposition 1** Independent of the part of immobile individuals \( \lambda \) in the economy, a symmetric equilibrium with identical fiscal policies \((t_j, g_j) = (t^*, g^*)\), identical rents \( r_j = \bar{r} \), and identical average incomes \( \bar{y}_j = \bar{y} \) in all jurisdictions \( j = 1, \ldots, J \) always exists.
4.2 An asymmetric equilibrium

Another possibility is that jurisdictions offer different tax-grant packages, i.e. \((t_j, g_j) \neq (t_h, g_h)\) for \(j, h = 1, \ldots, j-1, j+1, \ldots, J\) and \(j \neq h\). If \((t_j, g_j) \neq (t_h, g_h)\) mobile individuals may sort themselves into different jurisdictions: a phenomenon called "income stratification" in the literature. Stratification in this case depends on the part of the immobile population as well as the income inequality measured as the ratio between median and mean income.

To illustrate the possibility of such an asymmetric equilibrium, we focus for simplicity on the case of two jurisdictions with \((t_1, g_1) \neq (t_2, g_2)\). Suppose that jurisdiction \(j = 1\) is a low tax (wealthy) jurisdiction, whereas jurisdiction \(j = 2\) is a high tax (poor) jurisdiction. In this case jurisdiction \(j = 1\) must have a lower grant-rent differential, i.e. \((t_1, g_1 - r_1) < (t_2, g_2 - r_2)\), because otherwise all mobile individuals would like to live in the jurisdiction with low taxes and a high grant-rent differential.

In an equilibrium both jurisdictions must be populated and there is no extra-space. Thus, there must be a boundary individual with boundary income \(\tilde{y}\) who is just indifferent between the two jurisdictions. If an income stratification equilibrium exists, all mobile individuals with income \(y > \tilde{y}\) will live in the low tax (wealthy) jurisdiction \(j = 1\) and all mobile individuals with \(y \leq \tilde{y}\) will live in the high tax (poor) jurisdiction \(j = 2\). Equation (2) shows that if \(t_1 < t_2\) we must have \(\frac{y^m_1}{y^1_1} > \frac{y^m_2}{y^2_2}\). This implies that for stratification to occur income inequality must be higher in the low tax jurisdiction, as summarized by the following Lemma:

**Lemma 1** A necessary condition for stratification is that income inequality, defined as the ratio between median and mean income, is lower in the low tax jurisdiction.

In an income stratification equilibrium, the boundary individual with income \(\tilde{y}\) is the mobile individual with the lowest income in jurisdiction \(j = 1\) whereas in jurisdiction \(j = 2\) the individual with income \(\tilde{y}\) has the highest income among the mobile.

For the housing market to clear rents have to adjust such that exactly the fraction of wealthiest mobile individuals, \(x^M = 1 - F(\tilde{y})\), wants to live in jurisdiction \(j = 1\). Note that \(x^M\) depends on the immobile population \(\lambda\) in both jurisdictions as they determine
how many people can live in each jurisdiction. In the case of two jurisdictions half of the mobile individuals are considered wealthy while the other half are considered poor. Consequently, the fraction of wealthiest mobile individuals equals $x^M = \frac{1}{2}$ and the boundary income equals the economy’s median income, i.e. $\tilde{y} = F^{-1}(\frac{1}{2}) = y^m$.\footnote{Expressed differently, the fraction of wealthiest mobile individuals is equal to the remaining space in the low tax jurisdiction divided by the number of mobile from the economy’s population, i.e. $x^M = \frac{1-\lambda}{2-\lambda-\lambda} = \frac{1}{2}$.} Given the boundary income and a stratification equilibrium, the distribution of income, the mean and the median incomes in the two jurisdictions look as follows:

**Lemma 2** Let $\tilde{y}$ be the boundary income. In a stratification equilibrium the distributions of income in jurisdictions $j = 1$ and $j = 2$ are

\[
G_1(y) = \begin{cases} 
\lambda F(y) & y \leq \tilde{y} \\
(2 - \lambda)F(y) - (1 - \lambda) & y > \tilde{y}
\end{cases},
\]

\[
G_2(y) = \begin{cases} 
(2 - \lambda)F(y) & y \leq \tilde{y} \\
(1 - \lambda) + \lambda F(y) & y > \tilde{y}
\end{cases}.
\]

The mean incomes in each jurisdiction are

\[
\bar{y}_1 = \lambda \int_0^{\max_y} y f(y) dy + 2(1 - \lambda) \int_{\tilde{y}}^{\max_y} y f(y) dy,
\]

\[
\bar{y}_2 = \lambda \int_0^{\max_y} y f(y) dy + 2(1 - \lambda) \int_0^{\tilde{y}} y f(y) dy,
\]

and the median incomes in each jurisdiction are

\[
y_1^m = F^{-1}\left(\frac{13 - 2\lambda}{2 - \lambda} \right)
\]

\[
y_2^m = F^{-1}\left(\frac{1}{2 - \lambda} \right)
\]

All further proofs and detailed derivations are relegated to the appendix. Note that average incomes in $j = 1$ and $j = 2$ depend on the part of the immobile population...
\( \lambda \) and on the boundary income \( \tilde{y} \), forming the lower respectively the upper bound of the second integrals in (3) and (4). Median incomes (5) and (6) in both jurisdictions only depend on the part of the immobile population. Given a stratification equilibrium, increasing the part of immobile \( \lambda \) in the economy "crowds out" rich renters from \( j = 1 \) to \( j = 2 \) thereby increasing the median voter's income \( y_{2m}^{\lambda} \). Similarly, increasing \( \lambda \) in the poor jurisdiction allows, ceteris paribus, relatively poor renters to live in \( j = 1 \) thereby reducing \( y_{1m}^{\lambda} \).

As the mean and the median incomes in both jurisdiction depend on the part of immobile in the economy, equilibrium tax rates also depend on \( \lambda \), i.e. \( t_1^*(\lambda) = 1 - \frac{y_1^{\lambda}(\lambda)}{\tilde{y}(\lambda)} \) and \( t_2^*(\lambda) = 1 - \frac{y_2^{\lambda}(\lambda)}{\tilde{y}(\lambda)} \). By the government budget constraint the grant is determined as a function of \( \lambda \), i.e. \( g_j^*(\lambda) = (t_j^*(\lambda) - \frac{1}{2}t_j^2(\lambda)) \tilde{y}_j \). How will rents be determined in equilibrium? To see this consider the mobile individual with boundary income \( \tilde{y} \) who is indifferent between jurisdiction \( j = 1 \) and \( j = 2 \), i.e.

\[
V(t_1^*(\lambda), g_1^*(\lambda) - r_1, \tilde{y}) = V(t_2^*(\lambda), g_2^*(\lambda) - r_2, \tilde{y})
\]

\[
(1 - t_1^*(\lambda))\tilde{y} + g_1^*(\lambda) - r_1^* = (1 - t_2^*(\lambda))\tilde{y} + g_2^*(\lambda) - r_2^*.
\]

Note that \( r_2^* \) will be equal to the minimum rent \( r \) as absentee landlords only compete for renters in the high tax jurisdiction.\(^6\) Thus, we have

\[
r_1^* - r = (t_2^*(\lambda) - t_1^*(\lambda))\tilde{y} + g_1^*(\lambda) - g_2^*(\lambda) > 0
\]

This equation shows that a stratification equilibrium is fully determined by the part of the immobile population in the economy, \( \lambda \). Furthermore, it is logical that a stratification equilibrium can only exist, if the jurisdiction that we posited to be low tax (wealthy) in the beginning really exhibits lower taxes in equilibrium (a more detailed derivation can be found in the Appendix):

**Proposition 2** A stratification equilibrium exists if and only if \( t_2^*(\lambda) > t_1^*(\lambda) \), i.e. \( \lambda \) is

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\(^6\)Note that technically there is an infinitesimally small oversupply of houses such that rents in jurisdiction 2 are determined, similar to the symmetric case. For notational simplicity we omit it from the formulas.
such that the equilibrium tax rate in the poor jurisdiction is higher.

One might be tempted to think that there is always a stratification equilibrium for any $\lambda$: If taxes in jurisdiction $j = 1$ are higher than in jurisdiction $j = 2$, jurisdiction $j = 2$ might simply accommodate rich mobile individuals. This intuition is misleading, because the migration could change the outcome of the voting processes, such that taxes in jurisdiction 2 become higher relative to taxes in jurisdiction 1. Not every $\lambda$ can lead to a stratification equilibrium. The key intuition is that a low fraction of immobile people makes it more difficult for rich individuals to find a tax haven where inequality is low enough to achieve low tax rates in the voting process as shown in the following example:

**Example 1** Suppose $\lambda = 0$ (or close to zero), meaning that virtually everyone in the population is mobile and the distribution of income is uniform but different over three parts of the support, so that it is skewed to the right in a very stylized way:

$$f(y) = \begin{cases} 
\frac{1}{2} & \text{for } 0 \leq y \leq 1 \\
\frac{3}{8} & \text{for } 1 < y \leq 2 \\
\frac{1}{8} & \text{for } 2 < y \leq 3 
\end{cases}$$

There is some baseline inequality in the economy as $\bar{y} = \frac{9}{8} > y^m = 1$. Suppose individuals were stratified, such that the richest half lives in jurisdiction 1 and the poorest half lives in jurisdiction 2. The boundary income in this case is $\bar{y} = y^m = 1$. This implies for median incomes, mean incomes and thus for the tax rates in both jurisdictions:

$$y_1^m = \frac{5}{3}, \quad \bar{y}_1 = \frac{7}{4}, \quad t_1^* = \frac{1}{21}$$

$$y_2^m = \frac{1}{2}, \quad \bar{y}_2 = \frac{1}{2}, \quad t_2^* = 0$$

Thus, the tax rate in jurisdiction 2 (poor jurisdiction) is lower. Figure 1 Panel A shows the densities that would result for the two communities in this case. Intuitively the rich would like to go in jurisdiction 2, but if all the rich go in this jurisdiction the voting process will again yield a tax rate like in jurisdiction 1 now. The rich are unable to gather in one of the jurisdictions and agree about a low tax rate because the inequality
between them is higher than the inequality between the poor. For \( \lambda = 0 \) a statification equilibrium cannot exist.

**Figure 1 here**

Now suppose everything that one third of the population is immobile (\( \lambda = \frac{1}{3} \)). Under the same assumptions we obtain:

\[
\begin{align*}
    y_1^m &= \frac{23}{15}, \quad y_1 = \frac{37}{24} \Rightarrow t_1^* = \frac{1}{185} \\
    y_2^m &= \frac{3}{5}, \quad y_2 = \frac{17}{24} \Rightarrow t_2^* = \frac{13}{85}
\end{align*}
\]

Tax rates in the rich jurisdiction are now lower than in the poor jurisdiction so that nobody wants to change his location ex-post. Figure 1 Panel B shows the densities that would result for the two communities when \( \lambda = \frac{1}{3} \). The gray areas represent the part of the population that is not mobile and therefore those areas are the same in both communities. The resulting inequality after migration is lower in the rich jurisdiction and thus the tax rate will be lower there. Income stratification constitutes an equilibrium.

This example illustrates the potential role that mobility can play for the existence of income stratification equilibria. Hansen and Kessler (2001) noted that if jurisdictions do not have the same size and one jurisdiction is "small enough" then it is possible for the rich to gather there and agree on a tax rate that is lower than in the "big" jurisdiction, where inequality will be higher.\(^7\) Here, we note that even if the jurisdictions all have the same size, the fact that part of the population is not mobile can lead to income stratification.

If part of the population is immobile \( \lambda > 0 \), there is much more inequality in the jurisdiction where the poor decide to live, because some rich individuals there are not able to move. Those immobile rich will be exploited by the poor mobile taxpayers. For the mobile rich individuals the situation could possibly be even better than if everyone

\(^7\) The influence of the degree of mobility in our model can be compared to the effect of the geographical size in Hansen and Kessler (2001). They show that there is some threshold geographical size of a jurisdiction or country such that if there is at least one jurisdiction with this size or a lower size, a stratification equilibrium exists, independent of the precise distribution of income. Here, in this model, the distribution of income as well as the degree of mobility matters for the existence of a stratification equilibrium.
was mobile, because they can separate in a low tax jurisdiction and pay less taxes. The poor immobile living in the rich jurisdiction are probably also worse off, because they obtain less grants and have to pay higher rents. Finally, the general level of mobility is too low, stratification cannot arise, because rich individuals will be too unequal between each other relative to a the group of poor individuals.

4.3 The relationship between mobility and income stratification

An interesting question is now: For what values of the fraction of immobile in the population, $\lambda$, does a stratification equilibrium exist? Unfortunately, it is not possible to show for which $\lambda$ a stratification equilibrium exists without assuming a specific income distribution function. We propose to assume that income is distributed lognormally across individuals. This assumption implies that the distribution of incomes is bounded below and open above and that the biggest mass of individuals have relatively low incomes.\(^8\)

It turns out that the part of immobile individuals $\lambda$ in the economy and the economy’s income inequality both have a distinguishable effect on the existence of a stratification equilibrium. Given certain level of income inequality reflected by the parameter $\sigma$ for the lognormal distribution\(^9\), we can define a treshold $\tilde{\lambda}$ for the part of the immobile population implicitly:

**Proposition 3** For any given finite parameter $\sigma$ for the lognormal distribution, there is a unique threshold for the degree of mobility $\tilde{\lambda}$ implicitly defined by

$$t_2 - t_1 = \frac{1}{\exp \left( \frac{\sigma^2}{2} \right)} \left( \exp \left( \sigma \sqrt{2} \operatorname{erf}^{-1} \left( \frac{1-\lambda}{2-\lambda} \right) \right) - \frac{\exp \left( -\sigma \sqrt{2} \operatorname{erf}^{-1} \left( \frac{1-\lambda}{2-\lambda} \right) \right)}{\left[ \lambda + 2(1-\lambda) \Phi(\sigma) \right]} \right)$$

\(^7\)

\(^8\)As income distributions are usually skewed to the right the lognormal distribution has been found to approximate true income distributions quite closely and has been applied as a reasonably good characterization by other authors from the field (for example Epple and Romer 1991 and Hansen and Kessler 2001 among others).

\(^9\)Different measures of inequality are increasing in $\sigma$ for the lognormal distribution, for example the variance of income, $\text{Var}(y) = e^{\mu+\sigma^2}(e^{\sigma^2} - 1)$ or the ratio of mean income to median income, $\frac{\bar{y}}{y_{50}} = e^\frac{\sigma^2}{2}$. 

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If the part of the immobile population in the economy is lower than the threshold, i.e. \( \lambda < \tilde{\lambda} \), there is no stratification equilibrium. If the part of the immobile population is higher than the threshold, i.e. \( \lambda > \tilde{\lambda} \), there exists a stratification equilibrium, i.e. \( t_2 - t_1 > 0 \).

Figure 2, Panel A shows that there is a threshold level of part of the immobile population \( \tilde{\lambda} \) (here \( \tilde{\lambda} = 0.325 \) for \( \sigma = 0.5 \)) from which on the difference between the tax rates is positive and thus a stratification equilibrium exists. At the left of this threshold a stratification equilibrium cannot exist regardless in which of the two jurisdictions the rich move as the jurisdiction in which the rich have moved will feature a higher inequality and thus a higher tax rate.

If there is no stratification equilibrium, a natural question to raise is what happens instead? From a theoretical standpoint it is clear: There is simply no stable equilibrium in the metropolitan area.\(^{10}\) Empirically, we would observe migration that proceeds uninterruptedly and frequent changes in tax rates.

### 4.4 The role of overall inequality

The overall variance of the income distribution among individuals plays a crucial role for the threshold level \( \tilde{\lambda} \). Consider the four cases for \( \sigma \) in Panel B of Figure 2. The higher the initial variance in the income distribution the bigger is the range of possible values for the part of the immobile population from which on a stratification equilibrium arises. A numerical approximation of the threshold \( \tilde{\lambda} \), implicitly defined in Proposition 3, as a function of the variance confirms this finding as shown in Figure 3.\(^{11}\)

\(^{10}\)Decentralized political and locational choice may not produce a stable equilibrium solution. Stratification is not a general outcome of locational equilibrium models as contributions by Rose-Ackerman (1979), Epple and Platt (1998), Hansen and Kessler (2001) show. Westhoff (1977) constructs a model with a pure public good and shows that equilibria income sorting equilibria can exist. Nechyba (1997) states conditions for the equilibrium to be stratified.

\(^{11}\)The numerical approximation is done using a cubic-spline-approximation method. For more details on function approximation methods see Judd (1998) or Miranda and Fackler (2002).
In the figure we can see that as the variance $\sigma$ increases the threshold $\tilde{\lambda}$ becomes smaller, that is even a small fraction of immobile individuals is already sufficient to lead to a stratification equilibrium, if the overall variance in income is high. This can also be shown formally as summarized by the following proposition:

**Proposition 4** If $\sigma \to \infty$, there is always a stratification equilibrium independent of the part of the immobile population in the economy.

Intuitively, a higher overall variance $\sigma$ translates also into a higher variance for individual jurisdictions after migration as it also represents the variance in incomes in the immobile population which does not change. Because of the nature of the lognormal distribution median income is not affected by $\sigma$, but mean income is. Thus, inequality rises but simultaneously average incomes rise.\textsuperscript{12} Intuitively, it is easier for the "poor" to agree on a higher tax rate in the poor jurisdiction as here the mean has moved farther away from the median (i.e. inequality has risen more dramatically), when the overall variance is high. Moreover, in the rich jurisdiction, even a slightly lower tax rate has a relatively large effect on individual welfare of inhabitants, as they are "very rich" (higher mean income) compared to a situation where the overall variance is lower. But a slightly lower tax rate has a relatively small effect on the grants, because the tax base is larger, but not as much larger as the high incomes are. Thus, when the overall variance is high, the social loss from a lower tax rate is smaller than the individual gain from it. The result is that income stratification equilibria are more likely, when the overall mean income is high relative to the median (as measured by $\sigma$).

5 Conclusion

Differences in the degree of mobility between groups together with the overall regional inequality can have important implications on local redistribution in a federal system.\textsuperscript{12}For the lognormal distribution the mean is $\bar{y} = e^{\mu + \frac{\sigma^2}{2}}$ for a parameters $\mu$ and $\sigma$. 

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\textsuperscript{12}For the lognormal distribution the mean is $\bar{y} = e^{\mu + \frac{\sigma^2}{2}}$ for a parameters $\mu$ and $\sigma$. 

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Mobility itself is often seen as a constraint on local redistribution as it is usually believed to undermine local income redistribution in federal systems, because rich taxpayers can evade taxes by moving to "tax havens". Our main theoretical result suggests that even though out-migration possibilities may affect the potential for redistribution, income sorting in an asymmetric equilibrium (stratification) is not a general outcome and significant local redistribution may be feasible even when people are mobile.

We analyze local redistribution in a political economy model where local governments have tax-grant instruments, part of the population is mobile and voters are aware of migration effects of taxes and grants. This setting allows us to focus explicitly on the relationship between mobility, income inequality and redistribution. Our theoretical model leads to three predictions: (1) When inequality in the economy is high differences between inhabitants of local jurisdictions make stratification and thus low redistribution levels a likely outcome which is itself independent of the degree of mobility. (2) When part of the population is immobile, income sorting may arise as part of the rich immobile are exploited by the poor mobile while the rich mobile may escape taxes. For high levels of immobility stratification is less likely as only few people migrate and their influence on inequality is not sufficiently high. (3) Finally, despite high mobility, sorting may not arise as rich taxpayers are too unequal between each other.

Summarizing our results, we find that local income redistribution in a federal system is feasible if income inequality in the federation is not too high and mobility is not at an intermediate degree.

However, one has to be cautious when drawing immediate political conclusions from our results as we abstract certain institutional and political features of local jurisdictions given our single focus on redistribution and mobility. For example, we look at a purely redistributive setting instead of assuming a local public good which enters directly the utility function and we abstract from a housing market. Another, interesting extension to our current model would be to investigate a similar setting where mobility is endogenous and depends, for example, on income or wealth. Additionally, it would be interesting to include the long term decision of buying a house and thereby including possible capital gains and losses to the house due to capitalization of redistributive policies. Furthermore
it would be very interesting to investigate empirically the relationship between mobility and income stratification. All this is left to future research.

Appendix (Proofs)

Proof of Lemma 2

Distribution functions

In the jurisdiction 1, the low tax jurisdiction, only immobile individuals have an income below the boundary, so that for \( y \leq \tilde{y} \) we have:

\[
G_1(y) = \lambda F(y)
\]

There are two kinds of people with income above the boundary: (1) immobile with distribution \( \lambda F(y) \) and (2) mobile with distribution \( (1 - \lambda) \frac{F(y) - F(\tilde{y})}{1 - F(\tilde{y})} \) (the fraction of people that have income between \( \tilde{y} \) and \( y \) divided by the total fraction of people with income above \( \tilde{y} \)). Thus we have the following distribution for people with income \( y > \tilde{y} \):

\[
G_1(y) = \lambda F(y) + (1 - \lambda) \frac{F(y) - F(\tilde{y})}{1 - F(\tilde{y})} = (2 - \lambda) F(y) - (1 - \lambda).
\]

as \( x^M = 1 - F(\tilde{y}) = \frac{1}{2} \). The distribution of the population in jurisdiction 1 over income will thus have two different parts:

\[
G_1(y) = \begin{cases} 
\lambda F(y) & y \leq \tilde{y} \\
(2 - \lambda) F(y) - (1 - \lambda) & y > \tilde{y}
\end{cases}
\]

Similarly, one can show for jurisdiction 2, the high tax jurisdiction:

\[
G_2(y) = \begin{cases} 
(2 - \lambda) F(y) & y \leq \tilde{y} \\
(1 - \lambda) + \lambda F(y) & y > \tilde{y}
\end{cases}
\]
Mean incomes

In the low tax jurisdiction, jurisdiction 1, all immobile people have a mean income given by the original mean income, \( \int_0^{\text{max} y} y f(y) dy \) and all mobile people have a higher mean income given by \( \int_y^{\text{max} y} y \frac{f(y)}{1 - F(y)} dy \) (where the density is just weighted by the total fraction of individuals with income higher than the boundary and the support is from the boundary to the maximum income). Thus, mean income in jurisdiction 1 is

\[
\bar{y}_1 = \lambda \int_0^{\text{max} y} y f(y) dy + (1 - \lambda) \int_y^{\text{max} y} y \frac{f(y)}{1 - F(y)} dy
\]

\[
= \lambda \int_0^{\text{max} y} y f(y) dy + 2(1 - \lambda) \int_y^{\text{max} y} y f(y) dy,
\]

and similarly for jurisdiction 2:

\[
\bar{y}_2 = \lambda \int_0^{\text{max} y} y f(y) dy + 2(1 - \lambda) \int_y^{\text{max} y} y f(y) dy.
\]

Medians

Principally there are four different cases for the relationship between median income and the income of the boundary mobile individuum:

1. \( y^m \geq \hat{y} \) in both communities,
2. \( y_1^m \geq \hat{y} \) and \( y_2^m \leq \hat{y} \),
3. \( y_1^m \leq \hat{y} \) and \( y_2^m \geq \hat{y} \),
4. \( y^m \leq \hat{y} \) in both communities.

We want to show that we must be in case 2 and that the medians are given by equations (5) and (6). To do this, we will proceed in the following order. First we will show that if we are in case 2 the medians must be equal to (5) and (6). Then, we will show that given those medians we must be in case 2.
Part 1
Suppose \( y_m^1 \geq \bar{y} \) and \( y_m^2 \leq \bar{y} \). Then the median in jurisdiction 1 is implicitly defined as:
\[
G_1(y_m^1) = (2 - \lambda) F(y_m^1) - (1 - \lambda) = \frac{1}{2}
\]
\[
y_m^1 = F^{-1}\left(\frac{1 - \lambda}{2 - \lambda}\right).
\]
Similarly for the median in jurisdiction 2 we then have:
\[
G_2(y_m^2) = (2 - \lambda) F(y_m^2) = \frac{1}{2}
\]
\[
y_m^2 = F^{-1}\left(\frac{1}{2} \cdot \frac{1}{2 - \lambda}\right)
\]
We proceed in the same manner.

Part 2
Suppose the medians are given by \( y_m^1 = F^{-1}\left(\frac{1 + 2\lambda}{2 - \lambda}\right) \) and \( y_m^2 = F^{-1}\left(\frac{1}{2} \cdot \frac{1}{2 - \lambda}\right) \). This implies
\[
y_m^1 = F^{-1}\left(\frac{1 - \lambda}{2 - \lambda}\right) \geq \bar{y} = F^{-1}\left(\frac{1}{2}\right) \quad \text{for all } \lambda \leq 1
\]
as \( \frac{3 - 2\lambda}{2 - \lambda} \geq 1 \) for all \( \lambda \leq 1 \) and \( F^{-1}(\cdot) \) is a monotonously increasing function. Similarly:
\[
y_m^2 = F^{-1}\left(\frac{1}{2} \cdot \frac{1}{2 - \lambda}\right) \leq \bar{y} = F^{-1}\left(\frac{1}{2}\right) \quad \text{for all } \lambda \leq 1
\]
as \( \frac{1}{2 - \lambda} \leq 1 \) for all \( \lambda \leq 1 \) and \( F^{-1}(\cdot) \) is a monotonously increasing function.

Proof of Proposition 2
Lemma 1 shows that if a stratification equilibrium exists, \( t^*_2(\lambda) > t^*_1(\lambda) \) must hold. It remains to be shown that if \( t^*_2(\lambda) > t^*_1(\lambda) \) then we will have a stratification equilibrium. The remainder of this proof closely follows Hansen and Kessler (2001). Suppose, we have \( t^*_2(\lambda) > t^*_1(\lambda) \). Note that we must have a lower grant-rent differential in jurisdiction 1, i.e. \( (t^*_1(\lambda), g^*_1(\lambda) - r^*_1(\lambda)) < (t^*_2(\lambda), g^*_2(\lambda) - r^*_2(\lambda)) \). Otherwise, if the grant-rent differential was bigger in jurisdiction 1, everyone would like to live in jurisdiction 1. As all the jurisdictions must be populated and there is no extra space in jurisdiction 1. There
would be an over-demand for living in jurisdiction 1 and the price of housing would rise there until the grant-rent differential becomes smaller than in jurisdiction 2 and some individuals agree to live in jurisdiction 2.

What can be inferred about the distribution of mobile individuals over income in each jurisdiction? The ones that prefer a higher grant-rent to a lower tax rate will move to jurisdiction 2 while individuals preferring a lower tax rate to a higher grant-rent differential will move to jurisdiction 1. Totally differentiating shows that the slope of an indifference curve spanned by \( t \) and \( g - r \) is positive and increasing in income, i.e.

\[
\left. \frac{d(g_j - r_j)}{dt_i} \right|_{V=W} = y > 0.
\]

Thus, low-income individuals prefer regions with higher taxes combined with large grant-rent differentials, whereas high-income individuals prefer regions with low taxes and high grant-rent differentials. Consequently, from the fact that not everyone can live in jurisdiction, \((t_1^*(\lambda), g_1^*(\lambda) - r_1^*(\lambda)) < (t_2^*(\lambda), g_2^*(\lambda) - r_2^*(\lambda))\) and that the relative preference for lower taxes versus a higher grant-rent differential depends on income, we can conclude that a stratification equilibrium must arise. Individuals up to a certain boundary income \( y_1 \) live in jurisdiction 2 and individuals with higher income than the boundary \( y \) live in jurisdiction 1.

**Derivation of results of Example 1**

Using Lemma 1 and the distribution function given in the example we obtain for \( \lambda = 0 \):

\[
y_1^m = F^{-1}\left(\frac{3}{4}\right) = 1 + \frac{2}{3} = \frac{5}{3}, \quad \bar{y}_1 = 2 \int_1^2 \frac{3}{8}ydy + 2 \int_2^3 \frac{1}{8}ydy = \frac{7}{4}
\]

\[
\Rightarrow t_1^* = \frac{1}{21}
\]

\[
y_2^m = F^{-1}\left(\frac{1}{4}\right) = \frac{1}{2}, \quad \bar{y}_2 = 2 \int_0^{\frac{1}{2}} ydy = \frac{1}{2}
\]

\[
\Rightarrow t_2^* = 0
\]
and for $\lambda = \frac{1}{3}$:

$$y_1^m = F^{-1}\left(\frac{7}{10}\right) = 1 + \frac{4}{9} = \frac{23}{15}, \bar{y}_1 = \frac{1}{3}\bar{y} + \frac{4}{3}\left[\int_1^2 3ydy + \int_2^3 8ydy\right] = \frac{37}{24}$$

$$\Rightarrow t_1^* = \frac{1}{185}$$

$$y_2^m = F^{-1}\left(\frac{3}{10}\right) = \frac{3}{5}, \bar{y}_2 = \frac{1}{3}\bar{y} + \frac{4}{3}\int_0^1 2ydy = \frac{17}{24}$$

$$\Rightarrow t_2^* = \frac{13}{85}$$

**Proof of Proposition 3**

First we derive the equation that defines the threshold (equation (7)) and then we show that the threshold really exists and is unique. For the lognormal distribution we have $F(y) = \frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{\ln(y) - \mu}{\sigma\sqrt{2}}\right)$ by definition with an overall mean given by $\bar{y} = \exp\left(\mu + \frac{\sigma^2}{2}\right)$ and an overall median given by $y^m = \exp(\mu)$. We now calculate the means and the medians for this distribution.

**Means**

We can write

$$\int_{y^m}^{\infty} yf(y)dy = \exp\left(\mu + \frac{\sigma^2}{2}\right)\Phi(\sigma)$$

and

$$\int_0^{y^m} yf(y)dy = \exp\left(\mu + \frac{\sigma^2}{2}\right)(1 - \Phi(\sigma))$$

where $\Phi$ is cumulative distribution function of the standard normal. Thus equations (3) and (4) become

$$y_1 = \lambda\exp\left(\mu + \frac{\sigma^2}{2}\right) + 2(1 - \lambda)\exp\left(\mu + \frac{\sigma^2}{2}\right)\Phi(\sigma)$$

$$= \exp\left(\mu + \frac{\sigma^2}{2}\right)[\lambda + 2(1 - \lambda)\Phi(\sigma)]$$
and
\[ y_2 = \exp \left( \mu + \frac{\sigma^2}{2} \right) [\lambda + 2(1 - \lambda) (1 - \Phi(\sigma))] . \]

**Medians**

For the log normal distribution we obtain for \( y_m \):

\[ F(y_m^1) = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{\ln(y_m^1) - \mu}{\sigma \sqrt{2}} \right) = \frac{1}{2} \frac{1 - 2\lambda}{2 - \lambda}, \]

\[ y_1^m = \exp \left( \sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) + \mu \right) \]

and for \( y_m^2 \):

\[ F(y_m^2) = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{\ln(y_m^2) - \mu}{\sigma \sqrt{2}} \right) = \frac{1}{2} \frac{1}{2 - \lambda}, \]

\[ y_2^m = \exp \left( -\sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) + \mu \right) \]

**Difference between tax rates**

Using the means and medians, we can now calculate the difference between tax rates

\[ t_2 - t_1 = 1 - \frac{y_2^m}{y_2} - 1 + \frac{y_1^m}{y_1} = \frac{y_1^m}{y_1} - \frac{y_2^m}{y_2} \]

\[ = \frac{\exp \left( \sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) + \mu \right)}{\exp \left( \mu + \frac{\sigma^2}{2} \right) [\lambda + 2(1 - \lambda) \Phi(\sigma)]} \]

\[ - \frac{\exp \left( -\sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) + \mu \right)}{\exp \left( \mu + \frac{\sigma^2}{2} \right) [\lambda + 2(1 - \lambda) (1 - \Phi(\sigma))]}, \]

\[ = \frac{1}{\exp \left( \frac{\sigma^2}{2} \right)} \left( \frac{\exp \left( \sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \right)}{[\lambda + 2(1 - \lambda) \Phi(\sigma)]} \right) \]

This shows (7).

**Existence and uniqueness of stratification threshold**

We would like to show formally that there exists a unique stratification threshold for the case of two communities with equal immobility rates. We proceed in four steps:

1. Show that \( t_2 - t_1 = 0 \) at \( \lambda = 1 \) for all \( \sigma > 0 \).
2. Show that $t_2 - t_1 < 0$ at $\lambda = 0$ for all $\sigma > 0$.

3. Show that there must be at least one $\lambda$ for which $t_2 - t_1 = 0$ in the interval $\lambda \in [0, 1]$.

4. Show that there at most one $\lambda$ for which $t_2 - t_1 = 0$ in the interval $\lambda \in [0, 1]$.

Part 1 First, one can easily show that for any $\sigma$ the tax differential $(7)$ is equal to zero at $\lambda = 1$:

$$t_2 - t_1 = \frac{1}{\exp \left( \frac{\sigma^2}{2} \right)} \left( \exp \left( \sigma \sqrt{2} \text{erf}^{-1}(0) \right) \cdot 1 - \exp \left( -\sigma \sqrt{2} \text{erf}^{-1}(0) \right) \cdot 1 \right) = 0$$

Part 2 At $\lambda = 0$ we will have for $(7)$

$$t_2 - t_1 = \frac{1}{\exp \left( \frac{\sigma^2}{2} \right)} \left( \exp \left( \sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1}{2} \right) \right) 2 \left( 1 - \Phi(\sigma) \right) - \exp \left( -\sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1}{2} \right) \right) 2\Phi(\sigma) \right)$$

To see that this must be below zero for all $\sigma$ consider the numerator (substituting constant terms with $c = \sqrt{2} \text{erf}^{-1} \left( \frac{1}{2} \right) > 0$ for simplicity)

$$\exp (c\sigma) \left( 1 - \Phi(\sigma) \right) - \exp (-c\sigma) \Phi(\sigma)$$

$$= \exp (c\sigma) \left( 1 - \Phi(\sigma) \right) - \exp (-c\sigma) \left( 1 - \Phi(-\sigma) \right)$$

$$= H(\sigma) - H(-\sigma)$$

This shows that the numerator is the difference of a function $H(x) = \exp (cx) \left( 1 - \Phi(x) \right)$ at some point above zero with itself at some point below zero. Clearly if this function is
decreasing this difference must be negative. The derivative of this function is given by

\[
H'(x) = c \exp(cx)(1 - \Phi(x)) + \exp(cx)(-\phi(x)) \\
= \exp(cx)[c(1 - \Phi(x)) - \phi(x)]
\]

This derivative is negative because (1) the exponential term is positive and (2) the term in brackets is always negative. To see this consider

\[
c(1 - \Phi(x)) - \phi(x) < 0 \\
c = \sqrt{2} \text{erf}^{-1} \left( \frac{1}{2} \right) (\approx 0.67449) < \min \left[ \frac{\phi(x)}{(1 - \Phi(x))} \right] (\approx 0.79789) \leq \frac{\phi(x)}{(1 - \Phi(x))}.
\]

**Part 3** Consider the first derivative of the tax differential with respect to the immobility rate:

\[
\frac{\partial (t_2 - t_1)}{\partial \lambda} = \frac{1}{\exp \left( \frac{\sigma^2}{2} \right)} \left\{ \frac{\exp(k_1 q(\lambda)) [k_1 q'(\lambda)v_1(\lambda) - v'_1(\lambda)]}{v_1(\lambda)^2} - \frac{\exp(k_2 q(\lambda)) [k_2 q'(\lambda)v_2(\lambda) - v'_2(\lambda)]}{v_2(\lambda)^2} \right\}
\]

where we defined

\[
\begin{align*}
  k_1 &= -k_2 = \sigma \sqrt{2} \\
  q(\lambda) &= \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \\
  q'(\lambda) &= \frac{\sqrt{\pi}}{2} \exp \left( q(\lambda)^2 \right) \left( -\frac{1}{(2 - \lambda)^2} \right) \\
  v_1(\lambda) &= (1 - 2\Phi(\sigma))\lambda + 2\Phi(\sigma) \\
  v'_1(\lambda) &= (1 - 2\Phi(\sigma)) \\
  v_2(\lambda) &= (1 - 2\Phi(-\sigma))\lambda + 2\Phi(-\sigma) \\
  v'_2(\lambda) &= (1 - 2\Phi(-\sigma)).
\end{align*}
\]
At $\lambda = 1$ this first derivative is equal to

\[
\frac{1}{\exp \left( \frac{\pi^2}{4} \right)} \left\{ \frac{\exp(0) \left[ \sigma \sqrt{2} \left( -\frac{\sqrt{2}}{2} \right) - (1 - 2\Phi(\sigma)) \right]}{1} - \frac{\exp(0) \left[ -\sigma \sqrt{2} \left( -\frac{\sqrt{2}}{2} \right) - (1 - 2\Phi(-\sigma)) \right]}{1} \right\} = 0
\]

as

\[
\frac{2(\Phi(\sigma) - \Phi(-\sigma))}{\sigma} \leq \max \left( \frac{2(\Phi(\sigma) - \Phi(-\sigma))}{\sigma} \right) (\approx 1.6) < \sqrt{2\pi}(\approx 2.5).
\]

This means that there is at least one intersection with zero in the interval $[0, 1]$, because the tax differential function intersects at the point $\lambda = 1$ from above. From part 1 we know that at $\lambda = 1$ the tax difference $t_2 - t_1 = 0$ and from part 2 that at $\lambda = 0$ the tax difference is $t_2 - t_1 < 0$. This also implies that the number of intersections in the interval $[0, 1]$ has to be odd.

**Part 4** To see that there is a unique intersection expand (7) and set the numerator to zero

\[
\exp \left( \sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \right) \left[ \lambda + 2(1 - \lambda) (1 - \Phi(\sigma)) \right] - \exp \left( -\sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \right) \left[ \lambda + 2(1 - \lambda)\Phi(\sigma) \right] = 0
\]

\[
\exp \left( -\sigma 2\sqrt{2} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \right) - \frac{1 - (1 - \lambda) \text{erf} \left( \frac{\sigma}{\sqrt{2}} \right)}{1 + (1 - \lambda) \text{erf} \left( \frac{\sigma}{\sqrt{2}} \right)} = 0
\]

\[
m(\lambda) - n(\lambda) = 0
\]

Note that we have already shown that $n(0) > m(0)$ for any $\sigma$. If we can show that $n(\lambda)$ and $m(\lambda)$ are both convex increasing functions, then it is clear that they have at
most two intersections. See, Figure 4 as an illustration for $\sigma = 1$.

Together with part 3 of the proof, where we have shown that there must be at least one threshold, this would complete the proof that the threshold is unique.

That the function $n(\lambda)$ is a convex increasing function is easy to show:

\[
\frac{\partial n(\lambda)}{\partial \lambda} = \frac{2 \text{erf} \left( \frac{\sigma}{\sqrt{2}} \right)}{\left[1 - (1 - \lambda) \text{erf} \left( \frac{\sigma}{\sqrt{2}} \right) \right]^2} > 0 \text{ as } \text{erf}(x) > 0 \text{ for } x > 0
\]

\[
\frac{\partial^2 n(\lambda)}{(\partial \lambda)^2} = \frac{4 \text{erf} \left( \frac{\sigma}{\sqrt{2}} \right)}{\left[1 - (1 - \lambda) \text{erf} \left( \frac{\sigma}{\sqrt{2}} \right) \right]^3} > 0 \text{ as } \text{erf}(x) > 0 \text{ for } x > 0, (1 - \lambda) < 1
\]

and $\text{erf} \left( \frac{\sigma}{\sqrt{2}} \right) < 1$

The function $h(\lambda)$ is also clearly an increasing function:

\[
\frac{\partial m(\lambda)}{\partial \lambda} = \frac{\exp \left( \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \left( -2\sqrt{2}\sigma + \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \right) \right) \sqrt{2\pi} \sigma}{(2 - \lambda)^2} > 0
\]

But at first glance it is not clearly convex or concave:

\[
\frac{\partial^2 m(\lambda)}{(\partial \lambda)^2} = \frac{1}{(\lambda - 2)^4} \exp \left( \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \left( 2\sqrt{2}\sigma + \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \right) \right) \sqrt{2\pi} \sigma
\]

\[
\left[ 4 + \exp \left( \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right)^2 \right) \sqrt{2\pi} \sigma - 2\lambda - \exp \left( \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right)^2 \right) \sqrt{\pi} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \right]
\]

The term in square brackets determines, if this second derivative is positive or negative, because all other terms are positive.

As a preliminary consider that on the interval $\lambda \in [0, 1]$ we have:

\[
0 \leq \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) < \frac{1}{2}
\]

\[
0 \leq \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right)^2 < \frac{1}{4}
\]

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Now we want to show that the square brackets are positive. We split the square brackets into two terms $p_1(\lambda)$, the positive part, and $-p_2(\lambda)$, the negative part:

$$p_1(\lambda) = 4 + \exp \left( \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right)^2 \right) \sqrt{2\pi} \sigma$$

$$p_2(\lambda) = 2\lambda + \exp \left( \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right)^2 \right) \sqrt{\pi} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right)$$

where we will show that $p_1(\lambda) > p_2(\lambda)$, which means that the term in square brackets must be positive. Instead of comparing $p_1(\lambda)$ and $p_2(\lambda)$ we can also compare the lower bound of $p_1(\lambda)$ with respect to $\lambda$ on the given interval: $\min_{\lambda} [p_1(\lambda)] \leq p_1(\lambda)$ for all $\lambda \in [0, 1]$ with an upper bound of $p_2(\lambda)$ with respect to $\lambda$ on the given interval: $\max_{\lambda} [p_2(\lambda)] \geq p_2(\lambda)$ for all $\lambda \in [0, 1]$. Those boundaries are given by:

$$\min_{\lambda} [p_1(\lambda)] = 4 + \sqrt{2\pi} \sigma$$

$$\max_{\lambda} [p_2(\lambda)] = 2 + \exp \left( \frac{1}{4} \right) \frac{1}{2} \sqrt{\pi}$$

Comparing them we see that

$$4 + \sqrt{2\pi} \sigma > 2 + \exp \left( \frac{1}{4} \right) \frac{1}{2} \sqrt{\pi} (\approx 3.1379) \text{ for all } \sigma \geq 0$$

which completes the proof.

\[\blacksquare\]

**Proof of Proposition 4**

Consider the difference between tax rates:

$$t_2 - t_1 = \frac{1}{\exp \left( \frac{\sigma^2}{2} \right)} \left( \frac{\exp \left( \sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \right)}{[\lambda + 2(1 - \lambda)\Phi(\sigma)]} - \frac{\exp \left( -\sigma \sqrt{2} \text{erf}^{-1} \left( \frac{1 - \lambda}{2 - \lambda} \right) \right)}{[\lambda + 2(1 - \lambda) (1 - \Phi(\sigma))] \right)$$

We want to show that:

$$\lim_{\sigma \to \infty} (t_2 - t_1) = 0$$
Rewrite the tax difference as follows:

\[ t_2 - t_1 = \frac{\exp\left(\sigma^2 \text{erf}^{-1}\left(\frac{\sigma^2}{2}\right)\right)}{\lambda + 2(1 - \lambda)\Phi(\sigma)} - \exp\left(\sigma^2 \text{erf}^{-1}\left(\frac{\sigma^2}{2}\right)\right) \]

First note that the numerators limiting value is constant:

\[ \lim_{\sigma \to \infty} [\lambda + 2(1 - \lambda)\Phi(\sigma)] = 2 - \lambda \]
\[ \lim_{\sigma \to \infty} [\lambda + 2(1 - \lambda)(1 - \Phi(\sigma))] = \lambda \]

The second term thus goes to zero:

\[ \lim_{\sigma \to \infty} \frac{\exp\left(\sigma^2 \text{erf}^{-1}\left(\frac{\sigma^2}{2}\right)\right)}{\lambda + 2(1 - \lambda)(1 - \Phi(\sigma))] = \frac{\exp(-\infty)}{\lambda} = 0 \]

The first term also goes to zero because the square term dominates:

\[ \lim_{\sigma \to \infty} \frac{\exp\left(\sigma^2 \text{erf}^{-1}\left(\frac{\sigma^2}{2}\right)\right)}{\lambda + 2(1 - \lambda)\Phi(\sigma)} = \frac{\exp(-\infty)}{2 - \lambda} = 0 \]

Thus we must have:

\[ \lim_{\sigma \to \infty} (t_2 - t_1) = \lim_{\sigma \to \infty} \frac{\exp\left(\sigma^2 \text{erf}^{-1}\left(\frac{\sigma^2}{2}\right)\right)}{\lambda + 2(1 - \lambda)\Phi(\sigma)} - \lim_{\sigma \to \infty} \frac{\exp\left(\sigma^2 \text{erf}^{-1}\left(\frac{\sigma^2}{2}\right)\right)}{\lambda + 2(1 - \lambda)(1 - \Phi(\sigma))] = 0 \]

\[ \blacksquare \]

References


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Figure 1
Existence of Stratification Equilibria: A simple Example

A. Case $\lambda = 0$ (everyone mobile)

B. Case $\lambda = 1/3$ (one third of the population immobile)

Source: own representation
Figure 2
Relationship between the degree of mobility and the difference between tax rates

A. Case $\sigma = 0.5$

B. Cases $\sigma \in \{0.5; 1; 1.5; 2\}$

Source: own representation

The first part shows the difference between tax rates, $t_1-t_2$, as a function of the homeownership rate, $\lambda$, for a variance parameter, $\sigma = 0.5$. The second part shows the same function for different values of the variance parameter.
The figure shows the threshold immobility rate, from which on a stratification equilibrium occurs depending on the variance parameter. This function has been numerically approximated using the Collocation Method from the toolbox of Miranda and Fackler (2002) on the basis of cubic splines.

Source: own representation
Figure 4
Proof of unique Stratification Threshold: An Illustration

Source: own representation