A domain decomposition solver for ferromagnetism

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We present a simulation of ferromagnetic effects encountered in the electrolysis of aluminium. During the process of aluminium production, high densities of electric current are used to produce pure liquid aluminium out of aluminium oxide. These currents induce huge magnetic fields, which interact with the liquid aluminium, causing important flows inside the electrolytic cell. The forced flow may ultimately develop instabilities, causing damages to the electrolytic cell. To predict the aluminium flow, magnetic forces acting on the fluid must be calculated. Taking into account the effect of induction (Biot-Savart's law) is not sufficient, we must include also the ferromagnetic response of the metallic container of the electrolytic cell. Moreover, it is necessary to consider the non-linear behaviour of ferromagnetic material properties, which vary with the strength of the magnetic field.

The model of ferromagnetism includes elliptic PDE's for the potential of magnetic field, posed on the whole 3D space. Most standard solution techniques assemble and invert full matrices. In case of complicated industrial geometries, however, the order of these matrices is of order 10^5, reaching thus up-to-date limits in computer hardware and CPU time.

Instead, we propose an overlaping domain-decomposition technique, separating the simulation in the near-field adjacent to the electrolytic cell using finite elements, and the far-field using Green representation formulas. In this way, we circumvent the need to store and invert full matrices and permit to resolve industrial problems on standard computer equipment in a reasonable time. We estimate convergence speed and CPU/memory costs for our approach. We compare its performance to standard techniques on a simple testcase with 2 perpendicular plates subject to electric current in an infinite wire. Finally, we report results of simulations with non-linear ferromagnetic effects for the industrial testcase. The study has been sponsored by the Swiss CTI grant no. 6437.1 IWS-IW.