SA 2010 **Continuum mechanics I, Practice 1** Dr. Aleš Janka
Practice class of Oct 6, 2010

A straight beam (Fig 1 in black) is being deformed into a circular arc of diameter R (in red) according to the following rules:

- the mid-line of the beam is not stretched and the thickness of the beam is preserved,
- all sections perpendicular to the axis of the straight beam stay perpendicular to the mid-line after deformation.

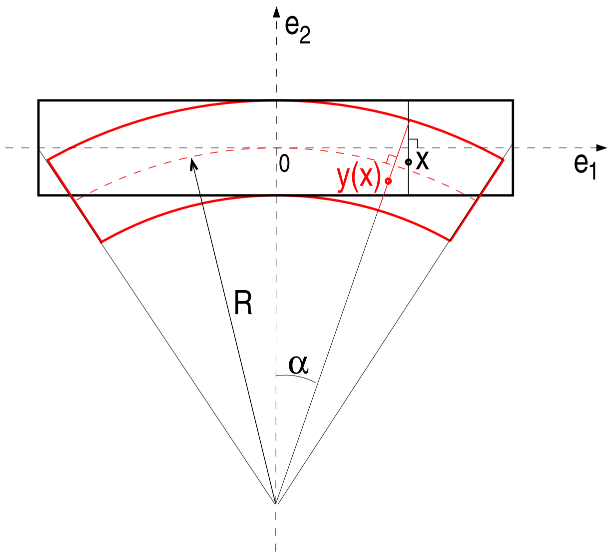

Figure 1: Deformation of a straight beam into a circular arc of diameter R

Let us work in cartesian coordinates, where $\vec{x} = x^i \vec{e}_i$ and $\vec{y}(\vec{x}) = y^j \vec{e}_j$.

) Check that the components y^j of $\vec{y}(\vec{x})$ are given by the following formula:

$$\begin{aligned} y^1 &= (R + x^2)\,\sin\left(\frac{x^1}{R}\right) \\ y^2 &= (R + x^2)\,\cos\left(\frac{x^1}{R}\right) - R. \end{aligned}$$

Calculate the inverse functions $x^1 = x^1(y^1, y^2)$, $x^2 = x^2(y^1, y^2)$.

a) Express analytically all components of the Green strain tensor ε_{ij}, $i, j = 1, 2$.

b) Express analytically all components of the Cauchy strain tensor e_{ij}, $i, j = 1, 2$.

c) Express analytically all components of the Almansi strain tensor $\mathcal{E}_{ij}$, $i, j = 1, 2$.

d) Inspired by the given Matlab code for calculating gradients of a scalar field over a finite-element mesh, write a Matlab function which gives the finite-element approximation of the above strain tensors.

e) Check the values of these strain tensors for examples of rigid-body motion (analytically or by Matlab-experimenting): do all rigid body modes produce always zero strain?