Material vs. local time derivative of a scalar field Φ

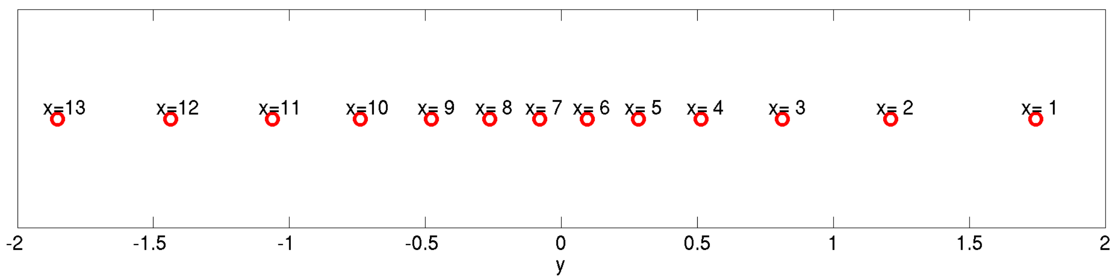
Physical scalar field Φ (e.g. concentration, density)

Lagrange formulation: on a material particle $\mathbf{x}$:

$$\Phi = \bar{\Phi}(\mathbf{x}, t) \qquad \frac{D\Phi}{Dt} = \frac{\partial \bar{\Phi}(\mathbf{x}, t)}{\partial t} \quad \textbf{material derivative}$$

Euler formulation: for observer in a fixed spatial point $\mathbf{y}$:

$$\Phi = \Phi(\mathbf{y}, t) \qquad \frac{\delta\Phi}{\delta t} = \frac{\partial \Phi(\mathbf{y}, t)}{\partial t} \quad \textbf{local derivative}$$



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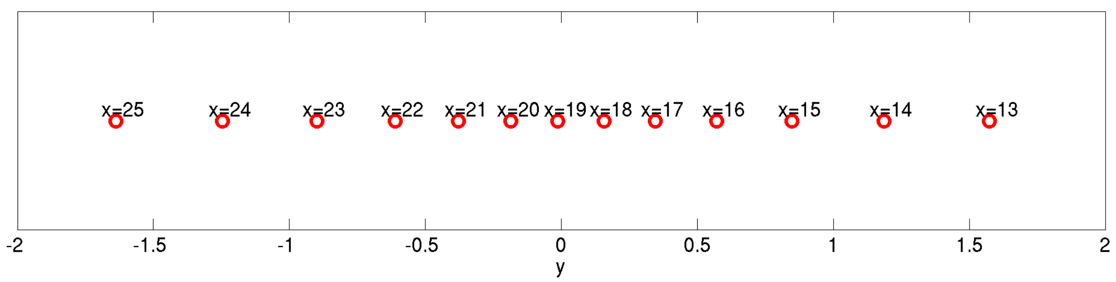

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Mutual relation: $\Phi(\mathbf{y}(\mathbf{x}, t), t) = \bar{\Phi}(\mathbf{x}, t)$:

$$\begin{aligned} \frac{D\Phi}{Dt} &= \frac{d}{dt}\bar{\Phi}(\mathbf{x}, t) = \frac{d}{dt}\Phi(\mathbf{y}(\mathbf{x}, t), t) \\ &= \frac{\partial \Phi}{\partial y^i}\frac{\partial y^i}{\partial t} + \frac{\partial \Phi(\mathbf{y}, t)}{\partial t} = \frac{\partial \Phi}{\partial y^i} v^i(\mathbf{x}, t) + \frac{\delta \Phi}{\delta t} \end{aligned}$$

where $v^i(\mathbf{x}, t) = \dfrac{\partial y^i}{\partial t}$ is the velocity of the particle $\mathbf{x}$.

Application of the material derivative: acceleration

Acceleration in Lagrange formulation:

$$a^k(\mathbf{x}, t) = \frac{Dv^k(\mathbf{x}, t)}{Dt} = \frac{\partial v^k(\mathbf{x}, t)}{\partial t}$$

Acceleration in Euler formulation:

$$a^k(\mathbf{y}, t) = \frac{\partial v^k(\mathbf{y}, t)}{\partial t} + \frac{\partial v^k(\mathbf{y}, t)}{\partial y^i} v^i$$